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SIMULATION OF HEAT TRANSFER IN A CONDUCTIVE HEAT EXCHANGER OF LOOP HEAT PIPES BY A FINITE-ELEMENT METHOD

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An analysis of a conductive heat exchanger used for joining links of coolant loop piping utilized in thermal regime systems of spacecraft is carried out. An indirect variant of the finite-element method specially modified for this case was used for the numerical solution. Temperature fields in the heat exchanger and dependences of the heat transfer coefficient on main parameters are obtained.

1. Presently, loop heat pipes (LHP) that provide heat transfer from operating spacecraft-borne equipment to a source of cold have found wide applications in systems of the thermal regime support of spacecraft. Heat exchangers of various types are used in this case. For example, an investigation of a radiation heat exchanger has been carried out in [1].

In order to ensure leak-proof LHP lines (when they must be joined), conductive heat exchangers consisting of two detachable boards are used. In one of the boards, the condenser of the first HP is situated, and the evaporator of the second HP is situated in the other one. The condenser and evaporator are most frequently manufactured in the form of a ring slot. The geometry of the heat exchanger makes it possible to consider the problem in a plane formulation. In view of that fact, and owing to the symmetry of the problem, the region of calculations has a rectangular shape with two semicircular notches (Fig. 1). External surfaces of the heat exchanger are heat-insulated, and therefore the value of the heat flux on them is considered to equal zero. The total heat flux is assumed to be known. Then, by setting up the temperature on the condenser (left notch) and the heat flux density on the evaporator, one can determine the heat transfer coefficient of the whole construction. The coefficient of thermal conductivity on the contact surface, whose value depends on a number of factors (pressure of board constriction, the degree of the finishing treatment of the surface, characteristics of the medium in the gap, etc.), is considered to be known. A similar method of calculation is presented in [2].

Thus, the mathematical formulation of the problem is as follows. It is necessary to find a solution of a steady-state heat conduction equation, i.e., the Laplace equation

$$\frac{\partial^2 T}{\partial x_i \partial x_i} = 0\,, \quad i = 1, 2\,, \tag{1}$$

under the following boundary conditions:

$$q = 0\,, \quad x \in \Gamma_1, \Gamma_3\,, \tag{2}$$

$$T = T_1\,, \quad x \in \Gamma_2\,, \tag{3}$$

$$q = q_0\,, \quad x \in \Gamma_4\,, \tag{4}$$

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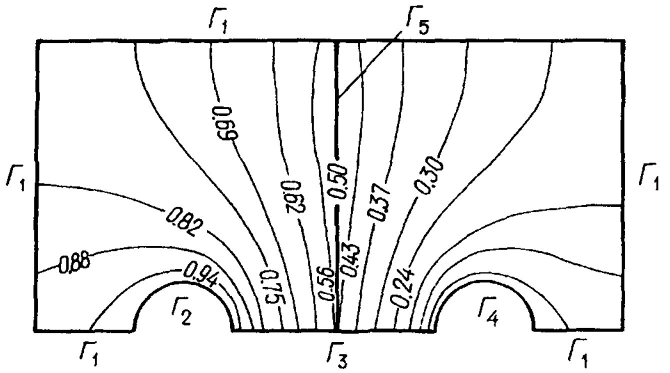

Fig. 1. Region of calculations and temperature distribution over the region for $R = 0.24$ and $U = 2$.

$$q_1 = q_2 = U(T_1 - T_2), \quad x \in \Gamma_5, \tag{5}$$

$$\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4.$$

Equation (1) with boundary conditions (2)-(4) and condition on the contact surface (5) are written in dimensionless variables. The board height and the temperature on the condenser wall are chosen as length and temperature scales.

2. An indirect variant of the finite-element method [3] was used to solve the problem. In doing this, we used a singular solution of the Laplace equation for the case of a single concentrated source:

$$T^*(x, \xi) = -\ln r/(2\pi), \tag{6}$$

$$q^*(x, \xi) = y_i n_i/(2\pi r^2). \tag{7}$$

We consider fictitious sources of unknown power to be distributed over the boundary of the region, including the contact surface. The left and right subregions will be treated separately, as independent zones interconnected by conditions on the contact (5), which consist in continuity of the heat flux and setting a temperature jump in accordance with the value of the thermal conductivity U. In view of the linear character of the problem, values of the temperature and flux at any point in the first and second zones can be found by convoluting the fundamental solutions (6), (7) with the corresponding distributions of the sources $\varphi_1(\xi)$ and $\varphi_2(\xi)$:

$$T_i(x) = \int_{S_i} \varphi_i(\xi) T^*(x, \xi)\, dS(\xi) + C_i, \quad x \in \Omega_i, \quad \xi \in S_i, \tag{8}$$

$$q_i(x) = \int_{S_i} \varphi_i(\xi) q^*(x, \xi)\, dS(\xi), \quad x \in \Omega_i, \quad \xi \in S_i. \tag{9}$$

where $i = 1, 2$ is the zone number. Constants C_i appear due to the logarithmic behavior of the fundamental solution for the temperature.

By passing to the limit $x \to x_0 (x_0 \in S_i)$ in (8), (9), we obtain a system of boundary integral equations equivalent to the boundary-value problem (1)-(5), and, by solving the system, we can find the distribution of the unknown intensities of the sources $\varphi_i(\xi)$:

$$T_i(x_0) = \int_{S_i} \varphi_i(\xi) T^*(x_0, \xi)\, dS(\xi) + C_i, \quad x_0 \in (S_i \cap \Gamma_5), \tag{10}$$

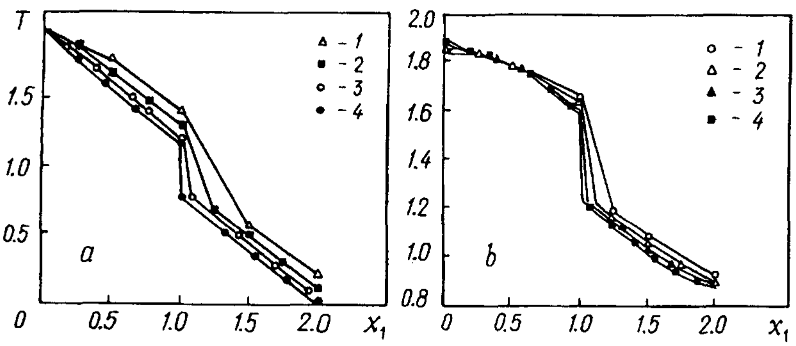

Fig. 2. Temperature distribution over the upper boundary of the region at different numbers of boundary elements compared with the analytical solution: a, 1) 16, 2) 32, and 3) 128 elements, and 4) analytical solution; b, 1) 46, 2) 54, 3) 140, and 4) 280 elements.

$$q_i(x_0) = -\frac{1}{2}\varphi(x_0) + \int_{S_i} \varphi_i(\xi)\, q^*(x_0, \xi)\, dS(\xi), \quad x_0 \in (S_i \cap \Gamma_5), \tag{11}$$

$$\int_{S_1} \varphi_1(\xi)\, q^*(x_0, \xi)\, dS(\xi) - \int_{S_2} \varphi_2(\xi)\, q^*(x_0, \xi)\, dS(\xi) - \frac{1}{2}[\varphi_1(x_0) - \varphi_2(x_0)] = 0, \quad x_0 \in \Gamma_5, \tag{12}$$

$$-\frac{1}{2}\varphi_1(x_0) + \int_{S_1} \varphi_1(x_0)\, q^*(x_0, \xi)\, dS(\xi) - U\left[\int_{S_1} \varphi_1(x_0)\, T^*(x_0, \xi)\, dS(\xi) + C_1 - \int_{S_2} \varphi_2(x_0)\, T^*(x_0, \xi)\, dS(\xi) - C_2\right] = 0, \quad x_0 \in \Gamma_5, \tag{13}$$

$$\int_{S_i} q_i(\xi)\, dS = 0. \tag{14}$$

The integral $\int_{S_i} \varphi_i(\xi) q^*(x_0, \xi) dS$ should be considered in the sense of the Cauchy principal value.

Equations (10) are used for portions of the outer boundary of the region $\Gamma \cap \Gamma_5$ where the temperature is set up, and Eqs. (11) are used for portions of the boundary where the heat flux is set up. Equations (14) guarantee the uniqueness of the solution obtained and in fact serve for evaluation of constants C_i.

To obtain an approximate solution of system (10)-(14), the region of calculations is divided into linear portions within the limits of which values of the functions $\varphi_i(\xi)$ are considered to be constant (constant boundary elements). The internal boundary is, in fact, taken into account twice: as a portion of the boundary of the first and second zones being considered, as has been noted, separately. The doubled number of unknown intensities of sources on the inner boundary $\varphi(\xi)$ corresponds to setting up two boundary conditions on it: continuity of flux (12) and a condition of the third kind (13). Upon discretization of relationships (10)-(14) in accordance with the specified scheme, we obtain a system of linear equations with the dimensions $(N_1 + N_2 + 2N_3 + 2) \times (N_1 + N_2 + 2N_3 + 2)$ for $(N_1 + N_2 + 2N_3 + 2)$ unknowns $\varphi(\xi)$. Coefficients of the system, which are given by integrals of functions (6), (7) over elements, can be calculated analytically in this case.

To solve the system, a modified Gauss method with the choice of the leading element was used to solve the system of equations. Values of the temperature and flux in internal and boundary points can be found from discrete analogs of Eqs. (8), (9) and (10), (11), respectively.

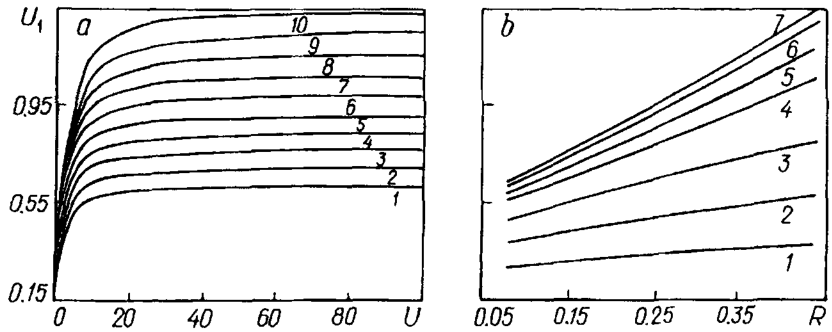

Fig. 3. Dependences of heat transfer coefficient of the heat exchanger on U and R: a, 1) $R = 0.08$, 2) 0.12, 3) 0.16, 4) 0.20, 5) 0.24, 6) 0.28, 7) 0.32, 8) 0.36, 9) 0.40, and 10) 0.44; b, 1) $U = 0.5$, 2) 1, 3) 2, 4) 5, 5) 10, 6) 30, and 7) 100.

3. In order to estimate the confidence of the results obtained, the numerical method was subjected to checks of various types. In particular, we solved a problem for a region consisting of two linked square subregions with thermoinsulated upper and lower boundaries. Temperatures were considered to be known on the left and right boundaries, and the temperature jump on the internal boundary was determined by the quantity U. For this, in fact one-dimensional problem, one can easily obtain an analytical solution, which was compared with the numerical results (Fig. 2a). The temperature distribution over the upper boundary was calculated at various numbers of boundary elements of equal length. When the boundaries are divided into 256 elements, the difference from the analytical solution is visually indistinguishable, and the value of the temperature jump (in this case $U = 2$) differs from the exact value of 0.4 only in the fourth digit. Figure 2b illustrates the good convergence of the method with an increasing number of boundary elements when solving the above-formulated problem for the region with ring-shaped notches. The character of the temperature distribution in the region of calculations is shown in Fig. 1. It should be noted that the equation of thermal balance is satisfied identically, since the discrete analogs of Eqs. (14) are satisfied exactly.

The total coefficient of thermal conductivity U_l (the inverse of the thermal resistance) is a main characteristic of the heat exchanger. The value of U_l, all other factors being the same, is determined by the value of the conductivity of the contact and outer radii of ring-shaped notches having the same shape as the condenser and evaporator. From the standpoint of design, the radii are usually chosen to be equal. Dependences of the thermal conductivity coefficient of the heat exchanger on U and R are presented in Fig. 3. These dependences calculated over the maximum possible range of variations of U and R can be used when designing the heat exchanger for evaluation of the heat transferred from the condenser to evaporator, which makes it possible to determine the functioning mode of the LHP. As is evident from Fig. 3a, beginning with $U \approx 20$, an improvement in the contact quality virtually does not affect the value of the heat exchanger's conductivity for arbitrary R (the maximum variation in U_l does not exceed 3%). At the same time, as follows from Fig. 3b, the value of U_l depends strongly on R for $U > 1$.

In summary, we should point out that a rather efficient finite element method based on the alternating directions scheme [5] has been used to solve this problem. A comparison with the boundary element method has substantiated that the former outperforms in both the accuracy and performance rate, which is almost an order of magnitude higher than in the case of the finite-element method.

NOTATION

T, temperature; x_i, Cartesian coordinates; q, heat flux density; Γ_i, portions of boundaries of the region of calculations; U, dimensionless conductivity coefficient of the contact surface of boards; q_1, q_2, T_1, and T_2, densities of heat flux and temperature on the contact surface on the side of the first and second subregions, respectively; $r^2 = (x_i - \xi_i)(x_i - \xi_i)$, distance between points x and ξ; $y_i = (x_i - \xi_i)$; n_i, components of the vector of the normal at the point X; $\varphi_i(\xi)$, intensity of fictitious sources; Ω_i, subregions with boundaries S_i; N_1, N_2, total number of elements of the first and second subregions; N_3, number of elements on the contact boundary; U_1, thermal conductivity coefficient of heat exchanger; R, external radius of ring notches.

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